Output factors for squared,
rectangular and elongated photon fields of linear accelerators:
A supplement to Stirling' formula.*

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Abstract.

For dose calculation in radiation therapy with photons a variety of basic beam data are required, such as percentage depth doses, dose profiles, and output factors (OF) for open as well as for wedged beams. In contrast to profile measurements, output factors have to be measured for all squared, rectangular and also for extremely elongated field sizes. To realize smooth output factor functions, at least 100 different field sizes with small increments in length and width have to be measured.

A simple modification of the Stirling' formula, taking into consideration the collimator exchange effect, allows an accurate prediction of the output factors for all field sizes, based on the experimental data survey for the minimum, the maximum and the reference field size (10 x 10 cm^2) only. The ratio of the calculated OF and the measured one's stays within 1.0025 ± 0.009, even for elongated and wedged field sizes. All major vendors of medical linear accelerators have been included in this study.

1. Introduction.

Dose calculation in radiation therapy requires an accurate beam modeling. This might increase the number of basic measurements dramatically, especially for the realization of a smooth output factor function, when the collimator exchange effect is taken into account. This is mandatory, because the output factor OF(F) is directly combined with the number of monitor units NMU to deliver a defined dose D(F, d, r) to a specific point:

$$D(F, d, r) = D_{cal} \cdot N_{MU} \cdot OF(F) \cdot TMR(F, d) \cdot OAR(F, r, d) \cdot (SCD/(SSD+d))^2 \cdot MOD, \qquad (1)$$

F	field size,	OF	output factor,
d	depth,	OAR	off axis ratio,
r	off axis distance,	SCD	source-calibration-distance,
D_{cal}	calibration factor,	SSD	source-skin-distance,
N_{MU}	number of MU,	MOD	dose modificator.

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The original Stirling' formula for calculation of equivalent squares does not take into account the collimator exchange effect, because the formalism was developed for a cobalt unit with a totally

The output factor itself is defined as the ratio of measured doses at depth d on central axis for a field size A x B and the reference field size 10 x 10 cm^2: "A" represents the distal (lower) collimator, "B" represents the proximal (upper) collimator, defining field width and field length, respectively. Forsquared field sizes a small increment in field width and length guarantees a smooth and accurate shape of the output factor function. But for rectangular and extremely elongated field shapes it is in addition of importance, which one of the jaws (collimators) defines length and width of the treatment field. The different influence of the jaws on output factor is called exchange effect and might amount up to 4 - 5 %, depending on construction features of the treatment head of the linear accelerator. [1,2]

different jaw and source construction compared to a linear accelerator. [3],[4],[5],[6],[7] And Co-60 units are not equipped with an ionization chamber for dose measurement, because this is accomplished by a redundant clock system with respect to dose rate, depending only on physical half life of the used nuclide.

Instead of introducing a treatment planning convention using always a specific jaw (upper or lower) for the larger field dimension, which is in practice hardly feasible, or increasing the number of OF – measurements, a simple correction factor is suggested here to take into account the jaw exchange problematic. This is known only for linear accelerators equipped with two pairs of movable jaws. [4]

2. Material, methods and procedures.

For determination of output factors in photon fields different commercially available medical linear accelerators of all major vendors (Elekta/Philips Siemens; Varian; Varian-Novalis) have been investigated. The photon energies of 12 units include 4-, 6-, 8-, 10-, 15- and 18 MeV-X, for a total of 30 beams. Only for two units output factors for wedged fields have been measured, because in general dynamic (or virtual) wedges are mainly used in daily routine.

Either a mini-phantom (build-up-cap), a block phantom (both made of PMMA) or a water phantom/ solid water phantom was used to measure relative doses at SSD of 90 cm or 100 cm. The diameter of the mini-phantom as well as the depths in phantom depend on the photon energy of the beam, or was set to d = 10 cm or to d = dmax, respectively. The field sizes varied between 2 x 2 cm^2 (4 MeV-X), 3 x 3 cm^2 (18 MeV-X) and 40 x 40 cm^2 for all linear accelerators, except for the stereotactic unit (6 MeV-X) with field sizes ranging from 1 x 1 cm^2 to 15 x 15 cm^2. (8) The field sizes for wedged beams where set according to the technical limitations of the particular linear accelerator, in most cases 4 - 20 cm in wedge direction. Readings where normalized to the reference field size 10 x 10 cm^2 : OF(10) = 1.000. For the stereotactic unit the output factors for wedged beams include the wedge factor : OF $<<$ 1. To cover the whole range of rectangular field sizes for most of the units the number of measured field sizes exceeds n = 80.

According to Sterling' formula the equivalent field sizes are calculated: $F = 2 \cdot A \cdot B / (A + B)$. The approximation of the output data for squared field sizes can be done by several analytical functions, for example a polynomial fit of higher order, a modified 2 - parameter exponential function [4], a sigmoidal 4 - parameter function (MMF model), or a so called Hoerl function (3 - parameter power function).[9] For this investigation the Weibull function $OF = a - b \cdot \exp(-c \cdot F^d)$, (OF: output factor; F: equivalent field size; a, b, c, d: constants) as well as the Hoerl model is used: $OF = a \cdot b^F \cdot F^c$ (OF: output factor; F: equivalent field size; a, b, c: constants). In this context the constants a, b, c, and d have no physical meaning: They where optimized for best approximation of the data. The shapes of this curves don't show any inflection points nor maxima or minima within the range of field sizes (Fmin… Fmax) and the slope is positive, describing the increase of the scattering volume of the flattening filter and the wedges, in case of the total output factors the additional scattering in the phantom as well. By means of the Marquardt-non-linear fitting algorithm [10] the parameter a, b, c, and d are determined for the squared field sizes.

The approximation is characterized in all cases by $\sigma = 0.0023$ ($\leq$ 0.3 %). Adding the output data of rectangular and elongated field sizes, recalculated by the Stirling' formula, increases the scattering of data significantly: $\sigma_J = 0.0093$ (J stands for a particular treatment unit in table 1). This is caused by the position of the proximal and distal collimator pair, and the influence of the build-in monitor chamber. To account for this collimator exchange phenomena, a simple correction, describing the different distances between the focal spot and the top of the movable collimator jaws, is introduced.[11,12] This data can be easily taken from the physical device description provided by the vendors:

$$F = 2 \cdot A \cdot C_1 \cdot B \cdot C_2 \cdot C_3 / (A \cdot C_1 + B \cdot C_2), \qquad (2)$$

A field width (lower collimator),

B field length (upper collimator),

C_1 correction for A,

C_2 correction for B,

C_3 re-normalization factor for corrected field size $[C_3 = (C_1 + C_2) / (2 \cdot C_1 \cdot C_2)]$.

Four effects mainly influence the output factor, and subsequently describe the collimator exchange factor: 1. scattering within the flattening filter ; 2. forward scattering of collimator jaws; 3. back scatter into the monitor chamber of the linear accelerator; 4. phantom scatter.[13] All of these effects show field size dependencies. But combined with correction factors C_1 and C_2 the projected primary collimator unveils the different geometrical position with respect to scattering into the internal dose monitor, which is positioned at 10 cm upstream of the top of the upper jaw collimator (see figure 1).

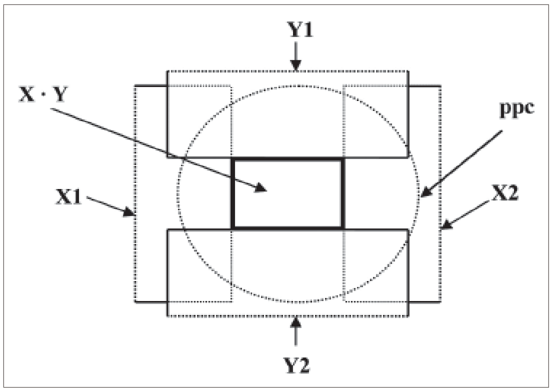

Figure 1. General design of collimators (beams-eye-view). X1, X2 : lower jaws; Y1, Y2 : upper jaws; X, Y : field size; ppc : projected primary collimator.

Out of this four above mentioned effects, only the backscatter (3) into the monitor chamber of the treatment unit and the forward scatter (2) to the external ionization chamber are influenced by the collimator exchange effect. A separation and estimation of these two effects is possible by evaluation of a complete set of OF data of a unit. This will be described in a future publication. Phantom scatter and scattering of the flattening filter are invariant according to the exchange effect, because the scatter defining volumes are identical in phantom and flattening filter, respectively.

3. Results.

The output data for squared field sizes of 31 photon beams from 12 different medical linear accelerators show extremely small deviations according to the used analytical function for approximation: $\sigma_{mean} = 0.0023$ ($\leq$ 0.3 %); in all cases the correlation coefficient $r > 0.999$. Including the data for rectangular and extremely elongated field sizes increases the scattering interval to about 2 … 3 %. This demonstrates clearly, that the Stirling' formula doesn't account for the exchange effect (figure 2).

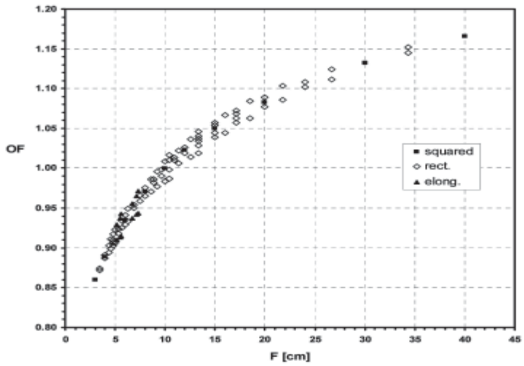

Figure 2. Output factors for squared, rectangular and elongated field sizes. F: equivalent field size; OF: output factor.

The application of the correction factors C1 and C_2 reduces this interval again to about 0.3 … 0.5 %. This uncertainty of data is comparable to the reproducibility of the measurements itself (figure 3). Slightly larger deviations occur in some special cases, when a motorized wedge was used for very small field sizes and the output measurements where done in the depth $d = d_{max}$ (linear accelerator unit P 6X and P 15X, P 6XW60 and P 15XW60) or when on a stereotactic unit the minimum field size seems to be inadequate small with respect to the used ionization chamber (linear accelerator O1 and O3) for the basic measurements : 1 x 1 cm^2 (table 1).

For demonstration of the quality of the above described correction formalism, the data of the linear accelerator J 8X are presented and analyzed more in detail. The best fit approximation off all measured squared field sizes (n = 10) is characterized by $\sigma_J = 0.0017$ ($\sigma_J \leq$ 0.2 %) and rJ = 0.999918.

Taking into account all measured field sizes (n = 100), squared, rectangular and elongated, increases the standard deviation to $\sigma_J = 0.0077$, and the regression coefficient reduces to $r_J = 0.9944$ (see also figure 2).

Table 1. Accuracy of prediction of OF

LinAc	E [MeV-X]	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	(Y/X)
E	4	0.996	0.999	0.001	1.004	1.002	3/40
A, B, J, K	6	0.992	1.000	0.003	1.000	0.995	2/40
A, J, K	18	1.012	1.011	0.002	1.000	1.003	3/40
J	8	1.003	1.005	0.002	0.999	0.997	3/40
F	6	0.996	1.001	0.001	1.017	1.013	3/40
G	10	1.000	1.003	0.001	1.010	1.010	3/40
G	15	1.008	1.009	0.002	1.010	1.016	3/40
B	10	1.005	1.003	0.003	0.995	0.997	2/40
C1	15	1.010	1.005	0.002	0.996	0.994	5/40
C2	15	1.008	-	0.002	-	1.002	5/35
CL*	18	1.005	1.014	0.002	1.000	0.995	4/35
O 1	6	1.020	1.000	0.003	1.005	1.017	1/40
O 2	15	1.009	1.009	0.005	1.003	1.009	3/40
O 3	6S	1.005	1.001	0.004	1.015	1.025	1/15
O W15	6	0.996	0.996	0.001	1.002	1.003	4/20/40
O W30	6	0.990	0.993	0.003	0.997	0.995	4/20/40
O W45	6	0.993	0.996	0.002	0.999	1.000	4/20/40
O W60	6	0.992	0.995	0.002	1.001	1.000	4/15/40
O W15	15	1.001	0.999	0.002	1.005	1.006	4/20/40
O W30	15	0.998	0.996	0.002	0.999	1.001	4/20/40
O W45	15	0.996	0.995	0.002	1.000	1.000	4/20/40
O W60	15	0.997	0.995	0.001	1.003	1.003	4/15/40
P	6	1.000	0.994	0.003	1.023	1.023	4/40
P W60	6	0.972	0.980	0.003	1.017	0.995	4/30/40
P	15	1.003	0.995	0.003	1.027	1.032	4/40
P W60	15	0.983	0.976	0.004	1.030	1.029	4/30/40
	μ	**1.000**	**1.000**	**0.0023**	**1.005**	**1.005**	
	σ	**0.009**	**0.007**	**0.0009**	**0.009**	**0.011**	

Δ_1 : Y = min; X = max; Δ_2 : Y = min + 1 cm; X = max; Δ_3 = σ Y = X= min … max;
Δ_4 : Y = max; X = min + 1 cm; Δ_5 : Y = max; X = min; (Y/X): minimum and maximum settings of movable jaws ; tab. values: calc./exp; * data from Purdy (1983).

The application of the exchange correction factors C_1 and C_2 minimizes the scattering of the output factors again: σ_J = 0.0019 (see figure 3). Even for the extremely elongated field sizes 3 x 40 cm^2 and 40 x 3 cm^2 an accurate prediction of the output factor is possible: the deviations between the measured and the predicted data are $\Delta_J \leq$ 0.2 % (see table 2). Because of the characteristics of both of the functions used for approximation, representing the construction features of the treatment head, the number of input data can be reduced to the minimum, the maximum and the reference field size. The results, based on this 3 – point - approximation, are also listed in table 2.

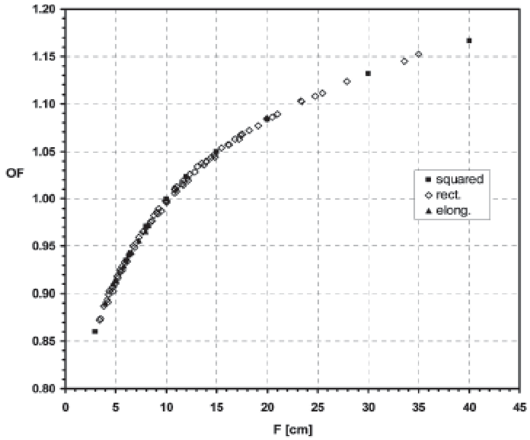

Figure 3. Output factors for squared and corrected rectangular and elongated field sizes. F : equivalent field size; OF : output factor.

4. Discussion and conclusion.

The collimator exchange effect already exceeds the 1% - level at moderate field sizes of about 20 x 20 cm^2, and for extremely elongated field sizes, such as 40 x 4 cm^2, which are sometimes in use for dorsal spine treatments, this phenomena introduces an uncertainty of more than 3% on particular medical linear accelerators. Beside this uncertainty a variety of other facts might influence the result of the dose calculation. According to propagation of errors even this small influence of the collimator exchange effect should be minimized. This can be accomplished by measuring the whole range of field sizes (n ≥ 100), setting the increment of length and width as small as necessary to get smooth shaped output factor functions, to minimize interpolation errors.

Or to use a simple algorithm, based on physical device data, some basic assumptions about involved scatter volumes, and only three field size measurements, which allow a prediction of output factors for the whole range of field sizes, open as well as wedged, with an accuracy of about 0.5 %, even for extremely elongated fields sizes and also accounting for the collimator exchange effect. This proposed addition to the Stirling' formula fulfills in an ideal manner the simplification and reduction of measurements without any loss of accuracy, because it depends on pre-known geometrical data instead of post-optimization of experimental results. [(14),(15),(6)]

Also for QA reasons this finding validate the assumption, that 3 field sizes characterize the whole OF data matrix of at least 100 single field measurements in an adequate and sufficient way.

Table 2. Prediction of OF for radiotherapy unit J8

F [cm^2]	squared[1]	all (uncorr.)[2]	all (corr.)[3]	3-pt.[4]	OF_{exp}[5]	$\Delta_{3\text{-pt.}}$ [%][6]
3 x 3	0.859	0.859	0.859	0.860	0.860	0.0
10 x 10	0.999	0.998	0.997	1.000	1.000	0.0
20 x 20	1.085	1.084	1.084	1.084	1.083	< 0.1
30 x 30	1.133	1.132	1.133	1.132	1.132	0.0
40 x 40	1.165	1.164	1.164	1.165	1.166	< 0.1
40 x 3	0.915	0.910	0.914	0.917	0.915	0.2
3 x 40	0.943	0.937	0.941	0.944	0.943	< 0.1
40 x 10	1.047	1.042	1.046	1.047	1.044	0.3
10 x 40	1.068	1.063	1.067	1.067	1.067	0.0
30 x 3	0.913	0.908	0.912	0.915	0.913	0.2
3 x 30	0.939	0.934	0.938	0.941	0.937	0.4
20 x 3	0.909	0.904	0.908	0.911	0.910	0.1
3 x 20	0.933	0.927	0.931	0.934	0.929	0.5
10 x 3	0.898	0.893	0.897	0.900	0.899	0.1
3 x 10	0.915	0.910	0.914	0.916	0.911	0.5

[1] all squared field sizes (n = 10); [2] all field sizes, including squared, rectangular and elongated (n = 100); [3] all field sizes corrected according to formula (2); [4] best-fit based on 3 field sizes (minimum, maximum and reference); [5] measured OF factors; [6] difference between 3-pt.-approximation and measured values of OF.

5. Addendum.

Experimentally it has been shown, that C_1 and C_2 work excellent to describe the collimator exchange effect on all linear accelerators of the main vendors:

$$F = 2 \cdot C_3 \cdot A \cdot C_1 \cdot B \cdot C_2 / (A \cdot C_1 + B \cdot C_2). \qquad (3)$$

A simple re-arrangement and some abbreviations result in a unique constant k, which is specific for each type of treatment unit, including open and wedged beams, because the principal field size dependence is described by the output factor function for squared field sizes:

$$F = A \cdot B \cdot (k + 1) / (k \cdot A + B), \qquad (4)$$

$$\text{with } k = C_1/C_2.$$

Table 3. Correction parameters.

Vendor	SIEMENS	Elekta/Philips	VARIAN/Novalis
k	1.333	1.329	1.315

In table 3 the values of k are listed for different vendors. The constant k, calculated by means of physical device data, seems to be identical to the analytically derived constant A, proposed in the literature. [6]

All commercially available medical linear accelerators show similar design characteristics with respect to the movable jaws and subsequently the factor k is comparable for all units.

When the correction factors C_1 and C_2 are identical, there is no exchange phenomena and the equivalent field size formula reduces to the well known Sterling' formula. Also for squared field sizes the correction function is neutral.

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